

Self-sensing of Damage and Strain in Carbon Fiber Polymer-Matrix Structural Composites by Electrical Resistance Measurement

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SUMMARY

The self-sensing of damage and strain in carbon fiber polymer-matrix structural composites has been attained by DC electrical resistance measurement. This paper reviews the self-sensing behavior, in addition to addressing the electrical contact configurations and providing a model for relating the resistance to the damage.

INTRODUCTION

Polymer-matrix composites for structural applications typically contain continuous fibers such as carbon, polymer and glass fibers, as continuous fibers tend to be more effective than the short fibers as a reinforcement. Polymer-matrix composites with continuous carbon fibers are used for aerospace, automobile and civil structures. Due to the fact that carbon fibers are electrically conductive, whereas polymer and glass fibers are not, carbon fiber composites exhibit electrical properties which depend on parameters such as strain and damage, thereby attaining the ability to sense themselves through electrical measurement. This self-sensing ability is valuable for smart structures, smart manufacturing, structural vibration control and structural health monitoring. Self-sensing means the elimination of attached or embedded sensors, as the structural material itself is the sensor. The consequence is reduced cost, enhanced durability, a larger sensing volume and an absence of mechanical property degradation.

This paper is focused on the use of DC electrical resistance measurement for the self-sensing of strain and damage in continuous carbon fiber polymer-matrix structural composites. Specifically, the quantity measured is the volume electrical resistance, which relates to the geometry-independent material property – the volume electrical resistivity. This

paper provides a review of self-sensing behavior, in addition to addressing the design of the electrical contacts (which are essential for the measurement) and providing a model for relating the electrical resistance to the type and amount of damage.

The use of experimental methods to study the mechanics of materials is increasingly widespread, due to advances in numerous experimental techniques and the scientific importance of experimental observations. These experimental methods should preferably be nondestructive, so that observation can be made in real time during mechanical deformation, damage infliction, temperature variation, humidity variation, etc. Real-time observation allows the study of both reversible and irreversible effects, whereas observation after the fact allows the study of irreversible effects only.

Electrical, optical and acoustic methods have all been used for experimental mechanics. Among these methods, electrical resistance methods are particularly attractive because of equipment simplicity and portability, applicability to materials or components of a large range of size, amenability to implementation in the field, fast response time, ability to provide directional information, and sensitivity to subtle microstructural changes (even those on the nanoscale). On the other hand, electrical resistance methods are limited to materials that are electrically conductive.

For example, carbon fiber epoxy-matrix composites are conductive.

SELF-SENSING BEHAVIOR REVIEW

Extensive damage, such as large cracks on the surface, can be sensed by visual inspection and liquid penetrant inspection. Both surface and sub-surface defects can be sensed by magnetic particle inspection (if the material is ferromagnetic), eddy current testing (if the material is electrically conducting and the defect is appropriately oriented with respect to the eddy current), ultrasonic testing (if the defect is appropriately oriented with respect to the ultrasonic wave propagation direction), x-radiography and other methods. However, these techniques tend not to be very sensitive to defects that are subtle in nature or microscopic in size. The resolution is particularly poor for magnetic particle inspection and x-radiography. The resolution tends to be better for ultrasonic methods, but it is still limited to about 0.1 mm (depending on the frequency of the ultrasonic wave). For effective hazard mitigation, it is important to be able to detect defects when they are small^{1,2}.

Recent interest in structural health monitoring has been centered on the use of embedded or attached sensors, such as piezoelectric, optical fiber, microelectromechanical, acoustic, dynamic response, phase transformation and other sensors^{3,4}, and the use of tagging through the incorporation of piezoelectric, magnetic or electrically conducting particles in the composite material⁵. The use of embedded sensors or particles tends to suffer from the mechanical property degradation of the composite. Furthermore, embedded sensors are hard to repair. Attached sensors are easier to repair than embedded ones, but they suffer from poor durability. Both embedded and attached sensors are much more expensive than the structural material, so they add significantly to the cost of the structure.

Electrical resistance measurements have received relatively little attention in relation to structural health monitoring. They do not involve any embedment or attachment, and so do not suffer from the problems described above for embedded or attached sensors. Furthermore, the method allows the entire structure to be monitored, whereas the use of embedded or attached sensors tend to allow the structure to be monitored at selected positions only.

Within a lamina with tows in the same direction, the electrical conductivity is highest in the carbon fiber

direction. In the transverse direction in the plane of the lamina, the conductivity is not zero, even though the polymer matrix is insulating, because there are contacts between fibers of adjacent tows⁶. In other words, a fraction of the fibers of one tow touches a fraction of the fibers of an adjacent tow here and there along the length of the fibers. These contacts result from the fact that fibers are not perfectly straight or parallel (even though the lamina is said to be unidirectional), and that the flow of the polymer matrix (or resin) during composite fabrication can cause a fiber to be incompletely covered by the polymer or resin (even though, prior to composite fabrication, each fiber may be completely covered by the polymer or resin, as in the case of a prepreg, i.e., a fiber sheet impregnated with the polymer or resin). Fiber waviness is known as marcelling. Thus, transverse conductivity gives information on the number of fiber-fiber contacts in the plane of the lamina.

For similar reasons, the contacts between fibers of adjacent laminae cause the conductivity in the through-thickness direction (direction perpendicular to the plane of the laminate) to be non-zero. Thus, the through-thickness conductivity gives information on the number of fiber-fiber contacts between adjacent laminae.

Matrix cracking between the tows of a lamina decreases the number of fiber-fiber contacts in the plane of the lamina, thus decreasing the transverse conductivity. Similarly, matrix cracking between adjacent laminae (as in delamination⁷) decreases the number of fiber-fiber contacts between adjacent laminae, thus decreasing the through-thickness conductivity. This means that the transverse and through-thickness conductivities can indicate damage in the form of matrix cracking.

Fiber damage (defect generation in the fiber, as distinct from fiber fracture) decreases the conductivity of a fiber, thereby decreasing the longitudinal conductivity (conductivity in the fiber direction). However, due to the brittleness of carbon fibers, the decrease in conductivity due to fiber damage prior to fiber fracture is rather small^{8,9}.

Fiber fracture causes a much larger decrease in the longitudinal conductivity of a lamina than fiber damage. If there is only one fiber, a broken fiber results in an open circuit, i.e., zero conductivity. However, a lamina has a large number of fibers and

adjacent fibers can make contact here and there. Therefore, the portions of a broken fiber still contribute to the longitudinal conductivity of the lamina. As a result, the decrease in conductivity due to fiber fracture is less than what it would be if a broken fiber did not contribute to the conductivity. Nevertheless, the effect of fiber fracture on the longitudinal conductivity is significant, so that the longitudinal conductivity can indicate damage in the form of fiber fracture¹⁰.

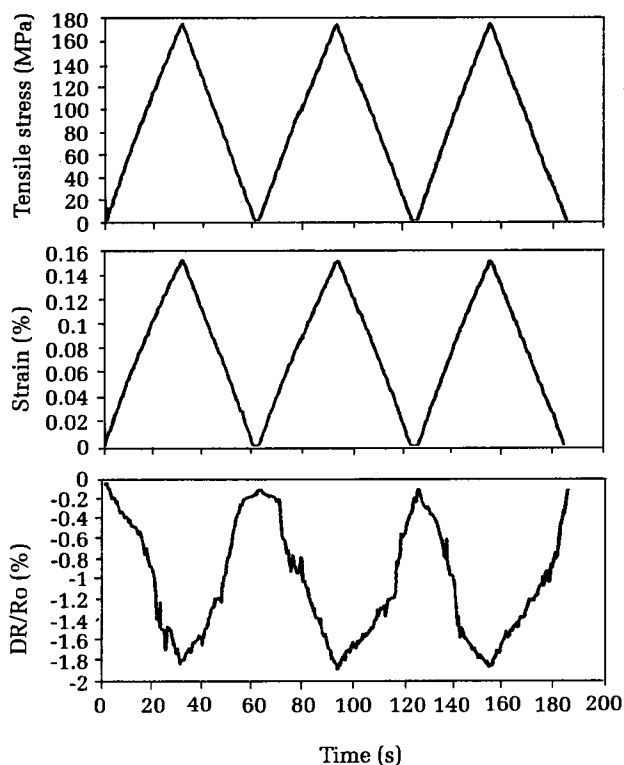
The measurement of the volume resistivity should be conducted by using the four-probe method, unless the specimen resistance is very high. In this method, two probes (the outer ones) are for passing current and two other probes (the inner ones) are for voltage measurement. In this way, the measured resistance excludes the contact resistance at the electrical contacts. On the other hand, the two-probe method involves only two probes, each of which serves both for passing current and for voltage measurement. Thus, the measured resistance in the two-probe method includes the contact resistance. The use of the two-probe method can give effects that are due to the contacts rather than the specimen itself, as shown for the effect of tension on the longitudinal resistance of a continuous carbon fiber epoxy-matrix composite¹¹. By using the four-probe method, the measured resistance decreases reversibly upon tensile loading, due to the decrease in the longitudinal resistivity of the composite upon tension. However, by using the two-probe method, the measured resistance increases reversibly upon tensile loading, due to the reversible loosening (degradation) of the electrical contacts.

The four electrical contacts in the four-probe method can be applied perimetrically at four planes that are perpendicular to the current direction. In cases where the specimen dimension is small in the current direction, such as measuring the through-thickness resistance of a laminate, each of the two current contacts can be in the form of a conductor loop (made by silver paint, for example) on each of the two outer surfaces of the laminate in the plane of the laminate. Each of the two voltage contacts can be in the form of a conductor dot within the loop^{7,12}. An alternate method is to make four of the laminae in the laminate extra long, so as to act as electrical leads¹³. The two outer leads are for current contacts and the two inner leads are for voltage contacts. The alternative method is less convenient than the one involving loops and dots, but it approaches more closely the ideal four-probe method. The use of a thin metal wire inserted

at an end into the interlaminar space during composite fabrication in order to serve as an electrical contact is not recommended, because the quality of the electrical contact between the metal wire and carbon fibers is hard to control and the wire is intrusive to the composite.

The simultaneous monitoring of damage (irreversible, whether due to stress or temperature, under static or dynamic conditions) and strain (reversible, due to stress under a dynamic condition) has been achieved in continuous carbon fiber polymer-matrix composites (both epoxy-matrix and thermoplastic-matrix composites), as the electrical resistance of the composite changes with damage and strain^{7,12,14-31}. Upon applying a longitudinal tension, the longitudinal resistivity decreases quite reversibly (Figure 1), primarily due to a strain-induced increase in the degree of fiber alignment. (This is consistent with the observed relationship between the fractional change in resistance of composites with different inherent degrees of fiber alignment and the tensile modulus

Figure 1 Longitudinal stress and strain and fractional resistance increase ($\Delta R/R_0$) obtained simultaneously during cyclic tension at a stress amplitude equal to 14% of the breaking stress for continuous fiber epoxy-matrix composite



under zero load, and also with that between the fractional change in resistance and the longitudinal resistivity under zero load). The through-thickness resistivity increases quite reversibly (Figure 2) for the same primary reason. Upon longitudinal compression, the longitudinal resistivity increases reversibly (Figure 3) while the through-thickness resistivity decreases reversibly, primarily due to a strain-induced decrease in the degree of fiber alignment and the consequent increase in the number of contacts between fibers of adjacent laminae.

That the degree of fiber alignment affects the resistivity of the composite in the longitudinal direction may be partly because the surface electrical contacts that supply current do not make direct electrical contact to the fibers throughout the cross-section of the composite. It may also be partly due to the presence of defects in some of the fibers. Upon longitudinal tension, all the effects occur in the reverse directions. These essentially reversible effects of strain provide mechanisms for sensing strain. On the other hand, slight matrix damage and/or disturbance to the fiber

Figure 2 Longitudinal stress and strain and the through-thickness $\Delta R/R_0$ obtained simultaneously during cyclic tension at a stress amplitude equal to 14% of the breaking stress for continuous fiber epoxy-matrix composite

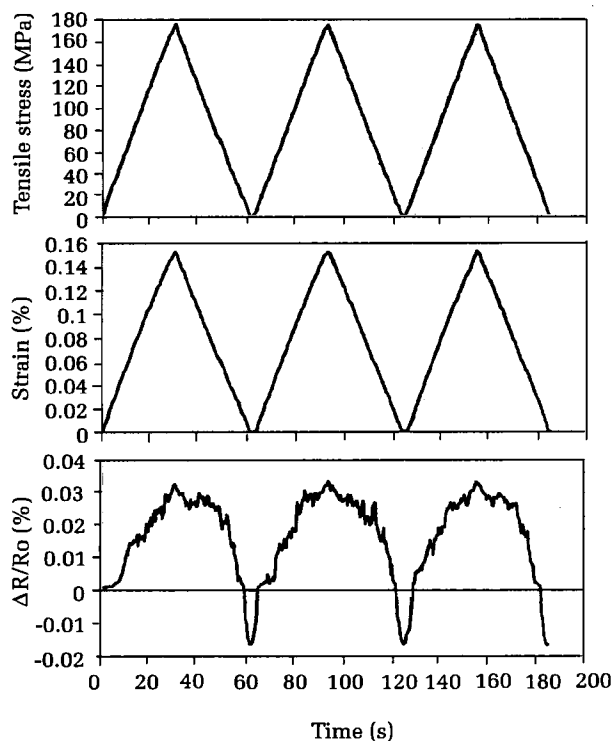
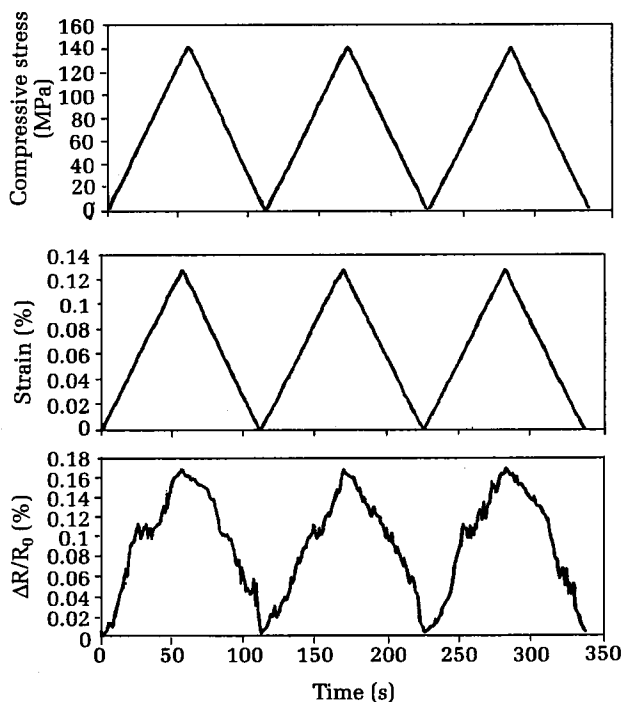


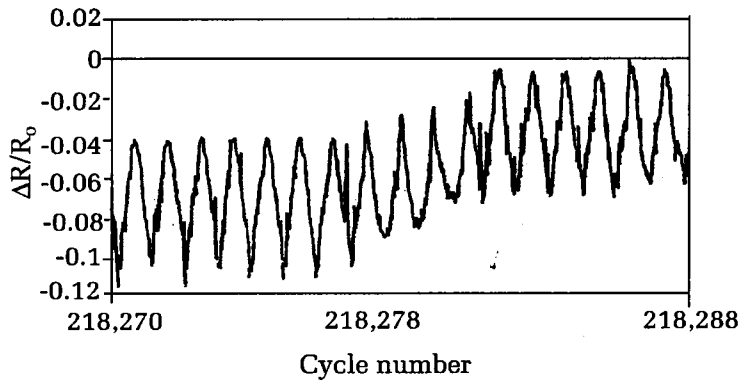
Figure 3 Longitudinal stress, strain and $\Delta R/R_0$ obtained simultaneously during cyclic compression (longitudinal) at a stress amplitude equal to 14% of the breaking stress for continuous fiber epoxy-matrix composite



arrangement is indicated by the longitudinal and through-thickness resistance decreasing irreversibly as a result of an increase in the number of contacts between fibers, as shown after one loading cycle in Figure 1 and 2. The irreversible decrease after one cycle is slight in Figure 1, but more significant in Figure 2. More significant damage in the form of delamination, interlaminar interface degradation or matrix damage is indicated by the through-thickness resistance (or, more exactly, the contact resistivity of the interlaminar interface) and the transverse resistance increasing irreversibly. Fiber breakage is indicated by the longitudinal resistance increasing irreversibly¹⁰.

During mechanical fatigue, delamination was observed from the through-thickness resistance to begin at 30% of the fatigue life, whereas fiber breakage (supported by the decrease in the secant modulus) was observed from the longitudinal resistance to begin at 50% of the fatigue life. Figure 4¹⁵ shows an irreversible resistance increase occurring at about 50% of the fatigue life during tension-tension fatigue testing of a unidirectional continuous carbon fiber epoxy-matrix composite. The resistance and stress

Figure 4 Variation of longitudinal $\Delta R/R_0$ with cycle number during tension-tension fatigue testing for a carbon fiber epoxy-matrix composite. Each cycle of reversible decrease in resistance is due to strain. The irreversible increase in resistance at around cycle no. 218,281 is due to damage in the form of fiber breakage



are in the fiber direction. The reversible changes in resistance are due to elastic strain, which causes the resistance to decrease reversibly in each cycle, as in Figure 1.

ELECTRICAL CONTACT DESIGN

Measurement of the volume electrical resistivity in both the longitudinal and through-thickness directions should use the four-probe method. This involves four electrical contacts – the outer two for passing current and the inner two for voltage measurement. The distance between the adjacent current and voltage contacts should be large enough so that the current density is uniform throughout the cross section in the region between the two voltage contacts. The minimum distance increases with increasing specimen thickness (i.e., increasing number of laminae) and with increasing through-thickness and transverse resistivities. It has been shown¹¹ that the two-probe method (i.e., the use of two contacts, each for both current and voltage) gives results which mainly reflect the quality of the electrical contacts rather than that of the composite specimen. Examples of the electrical contact configurations involving the four-probe method are shown below.

Figure 5 shows a configuration for measuring the overall longitudinal resistivity of a specimen. Each contact is on the surface, preferably around the whole perimeter on a plane perpendicular to the longitudinal direction. However, each contact may be just on the surface on one side only, as access to the bottom side of the composite component may be difficult in the field. If contacts on one side are used, the surface resistivity is measured rather than the volume resistivity. The surface resistivity is less meaningful scientifically than the volume resistivity. Contacts on one side are not as durable as perimetric contacts.

Figure 6 shows a configuration for measuring the through-thickness resistivity. This measurement involves current contacts in the form of loops (rectangular rather than circular in Figure 6) on two opposite sides and voltage contacts in the form of dots (rectangular rather than circular in Figure 6) on two opposite sides, such that each dot is within a loop. This contact design is because the thickness of the composite is too small for having current and voltage contacts on four different planes. Due to the lower resistivity in the longitudinal direction than in the through-thickness direction, the current path tends to bend outward (Figure 7), resulting in the effective

Figure 5 Electrical contact configuration for measuring the overall longitudinal electrical resistivity. I_1 and I_2 are current contacts. V_1 and V_2 are voltage contacts

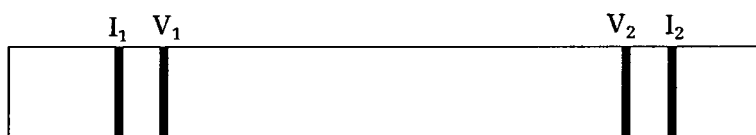


Figure 6 Electrical contact configuration for measuring the through-thickness resistivity. I_3 and I_4 (not shown, on the lower surface directly opposite to I_3) are current contacts and V_3 and V_4 (not shown, on the lower surface directly opposite to V_3) are voltage contacts for through-thickness resistivity measurement

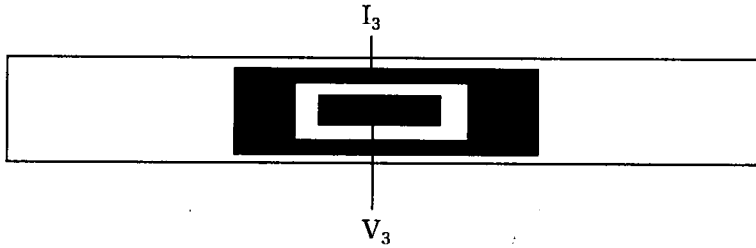
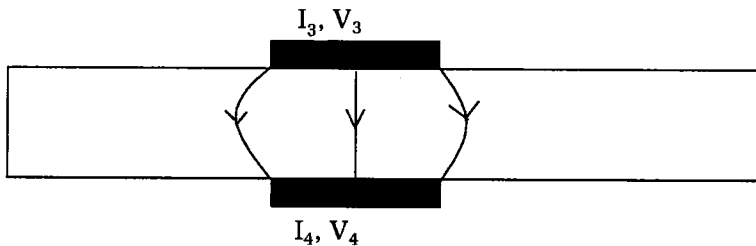


Figure 7 Edge view of the electrical contact configuration of Figure 6. The arrows indicate the current path for through-thickness resistivity measurement



cross-sectional area of the current path being larger than the area included by loop current contact, and hence giving rise to an underestimation of the through-thickness resistivity. All the contacts for through-thickness resistivity measurement are on one of two opposite surfaces.

Figure 8 shows a configuration for measuring the longitudinal resistivity distribution. The use of two current contacts (I_1 and I_2) and ten voltage contacts (V_1, V_2, \dots, V_{10}) allows the measurement of the longitudinal resistivity of nine segments (e.g., segment between V_1 and V_2 and segment between V_2 and V_3) and hence the resistivity distribution.

Figure 9 shows a configuration for measuring the through-thickness resistivity distribution. Comparison of Figure 8 and 9 shows that measurement of the longitudinal resistivity distribution involves fewer electrical contacts than that of the through-thickness resistivity distribution.

The application of electrical contacts does not require removal of the polymer matrix on the surface of the

composite. An electrically conductive paste such as silver particle paint has been successfully used^{6-13,15-19,31} for making the contacts. As silver paint is mechanically weak, the silver paint contact may be coated with a protective insulator such as epoxy.

DAMAGE MODEL

Previous work has emphasized experimental studies⁶⁻³⁵ rather than theoretical ones^{36,37}. A coherent approach that combines experimental and theoretical work is needed to interpret the resistance data in terms of the type and amount of damage.

A basic model of the composite is described in this section for a composite with N laminae. The model couples the volume electrical resistivity and the elastic modulus. The resistivity encompasses that in the through-thickness direction and that in the longitudinal direction. The through-thickness resistivity is increased by delamination and fiber-matrix debonding. The longitudinal resistivity is increased by fiber breakage and fiber-matrix debonding. Hence, the types of damage addressed include delamination, fiber-matrix debonding and fiber breakage.

Figure 8 Electrical contact configuration for measuring the longitudinal resistivity distribution. I_1 and I_2 are current contacts. V_1, V_2, \dots, V_{10} are voltage contacts. For example, the use of V_1 and V_2 gives the resistance between V_1 and V_2

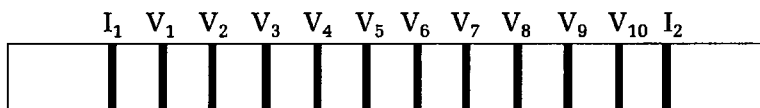
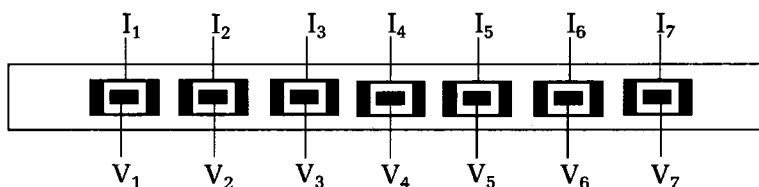


Figure 9 Electrical contact configuration for measuring the through-thickness resistivity distribution. The contacts are identical on the two opposite surfaces. Only those on one surface are shown



The model is aimed at experimental determination of the structural damage parameters, namely the average fraction of fibers broken in a lamina that involves fiber breakage (f_b), the average fraction of fibers debonded in a lamina that involves fiber-matrix debonding (f_m), the average fraction of fiber length debonded for the fibers that suffer from debonding (B), the number of laminae that involve fiber breakage (N_b), the number of interlaminar interfaces that suffer from delamination (N_d) and the average fraction of area that is delaminated for interlaminar interfaces that suffer from delamination (f_d). It should be noted that f_b and f_m constitute two subsets that overlap. The overlap corresponds to the fibers that suffer from both fiber breakage and fiber-matrix debonding. This overlap includes fibers that have fiber breakage and debonding occurring in the same locations as well as fibers that have breakage and debonding occurring in different locations. Because the model is aimed at experimental determination of the damage parameters, the number of damage parameters is kept small by the use of average quantities.

For the undamaged state, whatever is the lay-up configuration, the total resistance in the through-thickness direction is

$$R_{o,\text{total}}^t = NR_o^t + (N - 1)R_c, \quad (1)$$

where R_o^t is the volume resistance in the through-thickness direction for one lamina in the undamaged

state and R_c is the average contact resistance between adjacent laminae. Measurement of $R_{o,\text{total}}^t$ and use of Eq. (1) allows a determination of R_o^t .

The contact resistivity is the product of R_c and the contact area. It has been separately measured in prior work³²⁻³⁵; it is larger for unidirectional than crossply laminae³².

For the damaged state of a composite of any lay-up configuration that involves delamination only,

$$R_{\text{total}}^t = NR_o^t + (N - N_d - 1)R_c + \frac{N_d R_c}{1 - f_d}, \quad (2)$$

where N_d (an integer between 0 and N) is the number of interlaminar interfaces that have undergone some delamination and f_d is the average fraction of area that is delaminated for these interfaces.

For the damaged state of a composite of any lay-up configuration that involves fiber-matrix debonding, the volume resistance in the through-thickness direction of one damaged lamina is given by

$$R^t = (1 - f_m)R_o^t + \frac{f_m}{(1 - B)}R_o^t, \quad (3)$$

where f_m is the fraction of fibers that involve some fiber-matrix debonding in a lamina and B is the average fraction of the length of a partially debonded fiber that has undergone fiber-matrix debonding.

For the damaged state of a composite of any lay-up configuration that involves both delamination and fiber-matrix debonding,

$$R_{total}^t = NR^t + (N - N_d - 1)R_c + \frac{N_d R_c}{1 - f_d}, \quad (4)$$

where R^t is given by Eq. (3) and Eq. (2) is used.

For the undamaged state and a unidirectional composite, the total resistance in the longitudinal direction is

$$R_{o,total}^\ell = \frac{R_o^\ell}{N}, \quad (5)$$

where R_o^ℓ is the volume resistance in the longitudinal direction for one undamaged lamina. Measurement of $R_{o,total}^\ell$ allows determination of R_o^ℓ using Eq. (5).

For the damaged state of a unidirectional composite that involves fiber breakage, consideration of the laminae to be resistors in parallel gives

$$\frac{1}{R_{total}^\ell} = \frac{N - N_b}{R_o^\ell} + \frac{N_b}{R_b^\ell}, \quad (6)$$

where N_b (an integer between 0 and N) is the number of laminae that involve fiber breakage and R_b^ℓ is the longitudinal resistance of a lamina that involves fiber breakage. By considering a single lamina and its damaged and undamaged portions to be resistors in parallel,

$$\frac{1}{R_b^\ell} = \frac{1}{\left(\frac{R_o^\ell}{1 - f_b}\right)} + \frac{1}{\left(\frac{R_o^\ell}{f_b} + R_b^T\right)}, \quad (7)$$

where f_b is the fraction of fibers broken in a lamina that involves fiber breakage and R_b^T is the transverse volume resistance in the part of the lamina that involves fiber breakage. The term R_b^T is due to the resistance associated with the current crossing from a broken fiber to an adjacent one in the same lamina. Rearrangement of Eq. (7) gives

$$\frac{1}{R_b^\ell} = \frac{1 - f_b}{R_o^\ell} + \frac{f_b}{R_o^\ell + f_b R_b^T} \quad (8)$$

R_b^T is related to R_o^T (transverse volume resistance of the entire lamina in the undamaged state) by

$$R_b^T = f_b R_o^T \quad (9)$$

Combination of Eq. (8) and (9) gives

$$\frac{1}{R_b^\ell} = \frac{1 - f_b}{R_o^\ell} + \frac{f_b}{R_o^\ell + f_b^2 R_o^T} \quad (10)$$

Combination of Eq. (6) and (10) gives

$$\frac{1}{R_{total}^\ell} = \frac{N - N_b}{R_o^\ell} + N_b \left(\frac{1 - f_b}{R_o^\ell} + \frac{f_b}{R_o^\ell + f_b^2 R_o^T} \right) \quad (11)$$

For the damaged state of a unidirectional composite that involves both fiber breakage and fiber-matrix debonding, Eq. (8) becomes

$$\frac{1}{R_b^\ell} = \frac{1 - f_b}{R^\ell} + \frac{f_b}{R^\ell + f_b R_b^T} \quad (12)$$

where R^l is given by

$$R^\ell = (1 - B)R_o^\ell + \frac{B}{(1 - f_b f_m)} R_o^\ell, \quad (13)$$

The term $f_b f_m$ in Eq. (13) is due to the ineffective contribution of a broken and debonded fiber to electrical conduction in the longitudinal direction.

Because the fraction of fibers in a lamina that lie at the surfaces of the lamina is negligibly small, the effect of delamination on R_{total}^{ϵ} is negligible.

The elastic modulus of a unidirectional composite in the longitudinal direction is essentially not affected by fiber breakage, (in the absence of fiber-matrix debonding), unless the fiber fragments after the breakage are comparable to the critical length, which is typically of the order of 1 mm. The fiber breakage during typical damage of a composite in practical use is not that extensive within a given fiber. The longitudinal modulus is also essentially not affected by fiber-matrix debonding (in the absence of fiber breakage). However, the occurrence of fiber breakage and fiber-matrix debonding in the same location causes the debonded segments of the broken fiber to be ineffective for reinforcement, thereby decreasing the modulus.

Let E_o be the modulus in the undamaged state. Consider that the unidirectional composite is partly damaged, so that a lamina consists of columns in the longitudinal direction. Each column extends for the full length of the lamina. Columns that suffer from either fiber breakage or fiber-matrix debonding and those that suffer from both fiber breakage and fiber-matrix debonding, such that breakage and debonding do not occur in the same locations, are assumed to have modulus E_o . Columns that suffer from both fiber-matrix debonding and fiber-breakage such that debonding and fiber breakage take place in the same locations (modulus of column = E_{bm} , which is less than E_o) amount to a volume fraction of $k f_b f_m$, where k is a dimensionless constant between 0 and 1 (most likely 0.5-0.9). Columns that suffer from both fiber breakage and fiber-matrix debonding, such that the debonding and the fiber breakage do not take place in the same locations (modulus of column = E_o) amount to a volume fraction of $(1-k) f_b f_m$. The undamaged columns (modulus of column = E_o) amount to a volume fraction of $1 - f_b - f_m + f_b f_m$.

Consider a column that suffers from fiber breakage and fiber-matrix debonding in the same locations. For simplicity, consider that the column (diameter = d_m) consists of a single fiber (diameter = d_f) at the center of the column, such that it is symmetrically surrounded by the matrix. The length of the column is l_o . The length of the part of the column that suffers from fiber breakage and fiber-matrix debonding at similar locations is Bl_o . The elongation of the column under tension is

$$\Delta l = \frac{\sigma_{bm}}{E_o} l_o (1 - B) + \frac{\sigma_{m'}}{E_m} l_o B, \tag{14}$$

where the first term is the contribution from the undamaged portion of the column, the second term is the contribution from the damaged portion (which is sustained by the matrix only), s_{bm} is the stress on the column and s_{mc} is the stress on the matrix part of the column. The fiber volume fraction is given by

$$V_f = \frac{d_f^2}{d_m^2} \tag{15}$$

The longitudinal load is the same in the damaged and undamaged portions. Hence,

$$\sigma_{bm} \frac{\pi d_m^2}{4} = \sigma_{m'} \left(\frac{\pi d_m^2}{4} - \frac{\pi d_f^2}{4} \right) \tag{16}$$

Hence,

$$\sigma_{m'} = \frac{1}{1 - V_f} \sigma_{bm} \tag{17}$$

Using Eq. (17), Eq. (14) becomes

$$\Delta l = \frac{\sigma_{bm}}{E_o} l_o (1 - B) + \frac{\sigma_{bm}}{E_m (1 - V_f)} l_o B \tag{18}$$

From Eq. (18), the strain ϵ is given by

$$\epsilon = \frac{\Delta l}{l_o} = \frac{\sigma_{bm}}{E_o} (1 - B) + \frac{\sigma_{bm}}{E_m (1 - V_f)} B \tag{19}$$

Hence,

$$E_{bm} = \frac{\sigma_{bm}}{\epsilon} = \frac{E_o E_m (1 - V_f)}{E_m (1 - V_f) (1 - B) + E_o B} \tag{20}$$

Using the Rule of Mixtures for a composite with columns of modulus E_{bm} (volume fraction = $k f_b f_m$)

and columns of modulus E_o (volume fraction = $1 - kf_b f_m$), the modulus of the composite is

$$E = \frac{E_o E_m (1 - V_f)}{E_m (1 - V_f)(1 - B) + E_o B} kf_b f_m + E_o (1 - kf_b f_m) \quad (21)$$

Delamination is not considered in the derivation of Eq. (21). However, delamination has a negligible effect on the longitudinal tensile modulus, since the fraction of fibers of a lamina that are at the surfaces of the lamina is negligible.

The above model involves six damage parameters, i.e., f_b , f_m , f_d , B , N_d and N_b . Measurement of R'_{total} , R'_{total} , E_o and E and the use of Eq. (4), (11) and (21) allow determination of f_b , f_m and f_d , while B , N_d and N_b are estimated, as guided by optical examination.

CONCLUSIONS

The self-sensing of strain and damage in continuous carbon fiber polymer-matrix structural composites was attained by DC electrical resistance measurement. The strain is in the longitudinal direction and gives reversible changes in the resistivity. The damage can involve delamination, fiber-matrix debonding and fiber breakage and give rise to irreversible changes in the resistivity. The longitudinal and through-thickness resistivities and their distributions can be measured by using the four-probe method and interpreted in terms of the strain and damage distributions.

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